

Radiative Processes:

Nearly all of our knowledge about astronomical objects is obtained through the radiation that ^{they} produce. Their physical characteristics of their emission region are often inferred from features in their spectra. Since we understand radiation processes rather well, the theoretical modeling of these environments often leads to quantitative results.

Interactions of photons with charged particles are also important for other reasons. The same interactions that lead to radiation from high-energy charge particles also result in their energy loss. This will be important when discussing the spectrum of charged particles reaching us from high-energy sources.

Let us start by considering radiation from a moving charge.

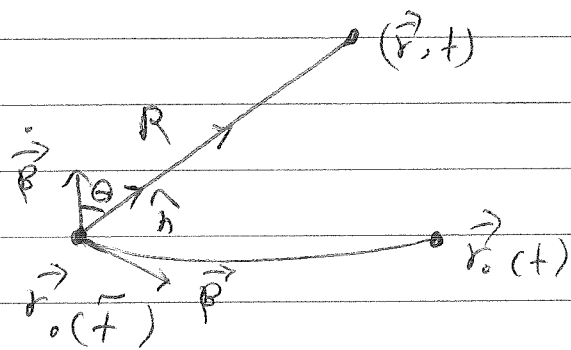
In the full relativistic treatment, the electric field of a particle moving with velocity $\vec{v}(t)$ is given by:

$$\vec{E}(\vec{r}, t) = \vec{E}_{\text{vel}}(\vec{r}, t) + \vec{E}_{\text{rad}}(\vec{r}, t)$$

$$\vec{E}_{\text{vel}} = q \left[\frac{(\hat{n} - \vec{\beta})(1 - \beta^2)}{k^3 R^2} \right]_{\hat{T}}, \quad \vec{E}_{\text{rad}} = \frac{q}{c} \left[\frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{k^3 R} \right]_{\hat{T}}$$

Here, we have:

$$\vec{\beta} \equiv \frac{\vec{v}}{c}, \quad k \equiv 1 - \hat{n} \cdot \vec{\beta}, \quad R = r - r_0$$



The expressions for \vec{E}_{vel} and \vec{E}_{rad} are calculated at the

retarded time \hat{T} , which is the solution to the following equation:

$$\hat{T} = t - \frac{|\vec{r}(t) - \vec{r}_0(\hat{T})|}{c}$$

We note that $|\vec{E}_{\text{vel}}|$, sometimes called the "velocity field", is

$\propto \frac{1}{R^2}$, while $|\vec{E}_{\text{rad}}|$ (called the "radiation field") is $\propto \frac{1}{R}$.

$\vec{E}_{\text{rad}} \neq 0$ only when $\dot{\vec{\beta}} \neq 0$, i.e., in the presence of particle

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acceleration. For example, when the particle is accelerated in a magnetic field (which results in synchrotron radiation), or in the electric field of another heavy charge particle (which leads to Bremsstrahlung radiation).

The magnetic field associated with \vec{E}_{rad} is given by:

$$\vec{B}_{\text{rad}}(\vec{r}, t) = \hat{n} \times \vec{E}_{\text{rad}}(\vec{r}, t)$$

Specializing to the non-relativistic limit, $\beta \ll 1$, and considering only \vec{E}_{rad} , we have:

$$\vec{E}_{\text{rad}} = \frac{q}{Rc^2} \hat{n} \times (\hat{n} \times \dot{\vec{v}})$$

$$|\vec{E}_{\text{rad}}| = |\vec{B}_{\text{rad}}| = \frac{q\dot{v}}{Rc^2} \sin\theta \quad (\theta \text{ is the angle between } \vec{R} \text{ and } \dot{\vec{v}})$$

The energy flux is given by the Poynting vector:

$$\vec{S} = \frac{c}{4\pi} \vec{E}_{\text{rad}} \times \vec{B}_{\text{rad}}, \quad S \equiv |\vec{S}| = \frac{c}{4\pi} E_{\text{rad}}^2 \quad (E_{\text{rad}} \equiv |\vec{E}_{\text{rad}}|)$$

The emitted power per given solid angle $d\Omega$ is:

$$\frac{dP}{d\Omega} = R^2 S = \frac{q^2 \dot{v}^2}{4\pi c^3} \sin^2\theta$$

Hence:

$$P = \int dP = \frac{2 q^2 \ddot{v}^2}{3c^3} \quad (\text{Larmor formula})$$

The point is that P depends on \ddot{v} , which is the second time

derivative of the electric dipole moment of the charge

\vec{d} . We therefore have:

$$P = \frac{2}{3c^3} \ddot{\vec{d}}^2 \quad \text{electric}$$

This is the familiar dipole approximation for a single non-relativistic particle.

Beside the total power, it is also important to know the

frequency distribution of the radiation. This can be done

by writing \vec{d} and \vec{E}_{rad} in the frequency domain (i.e.,

their Fourier transform):

$$\vec{d}(t) = \int_{-\infty}^{+\infty} e^{-i\omega t} \vec{d}(\omega) d\omega \Rightarrow \ddot{\vec{d}}(t) = - \int_{-\infty}^{+\infty} \omega^2 e^{-i\omega t} \vec{d}(\omega) d\omega$$

$$\Rightarrow \vec{E}_{\text{rad}}(t) = - \int_{-\infty}^{+\infty} \omega^2 e^{-i\omega t} \vec{d}(\omega) \frac{\sin\theta}{Rc^2} d\omega$$

Then;

$$\frac{dP}{dA} = \frac{c}{4\pi} E_{\text{rad}}^2(t) \Rightarrow \frac{d^2 W}{dt dA} = \frac{c}{4\pi} E_{\text{rad}}^2(t) \Rightarrow \frac{dW}{dA} = \frac{c}{4\pi} \int_{-\infty}^{+\infty} E_{\text{rad}}^2(t) dt$$

$$= \frac{c}{2} \int_{-\infty}^{+\infty} \bar{E}_{\text{rad}}^2(\omega) d\omega$$

Here we have used the Parseval's theorem $\int_{-\infty}^{+\infty} E_{\text{rad}}^2(t) dt = 2\pi \int_{-\infty}^{+\infty} \bar{E}_{\text{rad}}^2(\omega) d\omega$. Also, since $\bar{E}_{\text{rad}}^2(\omega)$ is an even function of ω , we find;

$$\frac{dW}{dA} = c \int_0^{+\infty} \bar{E}_{\text{rad}}^2(\omega) d\omega$$

In terms of the dipole moment in frequency domain, $\vec{d}(\omega)$, we have:

$$\frac{dW}{dA} = \frac{1}{R^2 c^3} \int_0^{+\infty} \omega^4 |\vec{d}(\omega)|^2 \sin^2 \theta d\omega$$

Thus;

$$\frac{dW}{d\omega d\Omega} = \frac{\omega^4}{c^3} |\vec{d}(\omega)|^2 \sin^2 \theta \Rightarrow \frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} |\vec{d}(\omega)|^2$$

We should not forget that this result is restricted to situations where the radiating particle is in the

non-relativistic regime.

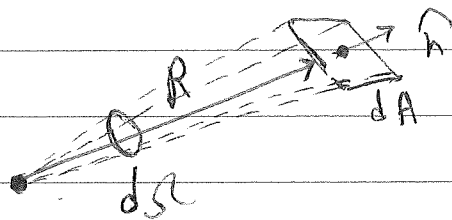
Intensity:

Before discussing various radiation mechanisms, it is useful to establish some basic definitions and see how one transforms a description of the photon field from one reference frame to another one.

The specific intensity at a frequency ν , denoted by I_ν , is defined by the relations

$$dW = I_\nu dA dt d\Omega d\nu$$

Here, dW is an element of energy in the frequency range $d\nu$ that crosses the area dA in time dt :



We note that the intensity represents the energy carried

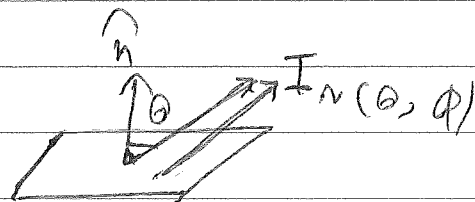
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along the direction of propagation, which in general may be in any direction relative to the unit vector \hat{n} normal to dA .

Therefore, the flux density is given by:

$$F_N = \int I_N(\theta, \varphi) \cos\theta \, d\Omega$$

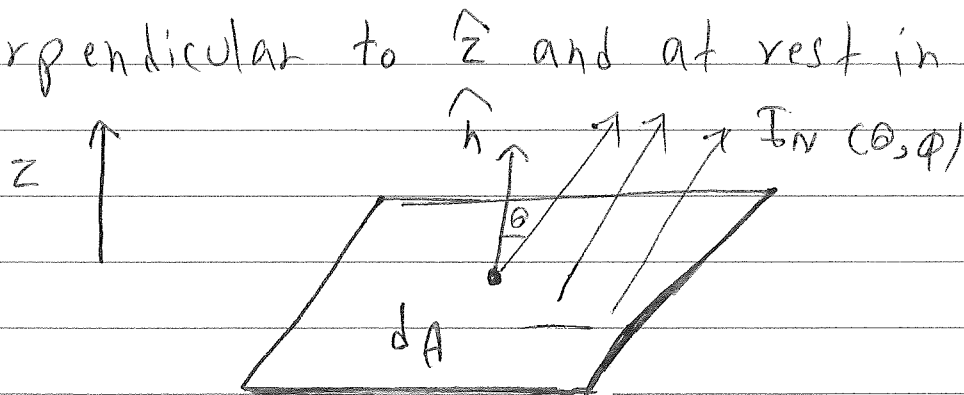
The total flux F and intensity I



are:

$$F = \int F_N \, d\Omega, \quad I = \int I_N \, d\Omega$$

Now let us consider the number of photons N in the frequency interval $(\nu, \nu + d\nu)$ that pass through an element of area dA perpendicular to \hat{z} and at rest in the lab:



$$dN = \left[\frac{I_N}{h\nu} \right] d\Omega \, d\nu \, dA \cos\theta \, dt$$

energy of an individual photon with frequency ν

We want to see what a second observer moving with velocity v in the z direction finds. The element of area moves with velocity $-v$ according to this observer, which results in an additional volume $v dA' dt'$ swept by the area. Hence:

$$dN' = \left[\frac{I_{\nu}'}{h\nu'} \right] d\Omega' d\nu' \left[dA' \cos\theta' dt' + \frac{v}{c} dA' dt' \right]$$

The total number of photons passing through the element must be the same according to both observers:

$$N = dN' \Rightarrow \left[\frac{I_{\nu}}{h\nu} \right] d\Omega d\nu \cos\theta dt = \left[\frac{I_{\nu}'}{h\nu'} \right] d\Omega' d\nu' (\cos\theta' + \frac{v}{c}) dt'$$

Here we have used $dA = dA'$ since the element is perpendicular to the relative velocity.

Different remaining quantities in above are related through Lorentz transformations as follows:

$$dt' = \gamma dt, \quad \cos\theta = \frac{\cos\theta' + \beta}{1 + \beta \cos\theta'}, \quad \frac{d\Omega}{\gamma^2 (1 + \beta \cos\theta')^2} = \frac{d\Omega'}{\gamma^2 (1 + \beta \cos\theta')^2}$$

$$v = \gamma (1 + \beta \cos\theta') v'$$

We therefore find:

$$\frac{I_\nu}{\nu^3} = \frac{I'_\nu}{\nu'^3}$$

We see that the quantity $\frac{I_\nu}{\nu^3}$ is a Lorentz-invariant quantity. This quantity has many uses in high energy astrophysics, including a direct application to the radiative flux produced in relativistic jets (which we will discuss later).